Vagueness, Haziness, and Fuzziness in Logic, Science, and Medicine – Before and When Fuzzy Logic Began

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1. Introduction

This contribution deals with developments in the history of philosophy, logic and mathematics before and when fuzzy logic began. Even though the term "fuzzy" was introduced by Lotfi Zadeh in 1964/65 it should be noted that older concepts of "vagueness" and "haziness" have been discussed in philosophy, logic, mathematics, applied sciences, and medicine. This paper delineates some specific paths through the history of the use of these "loose concepts" in science. The theory of fuzzy sets is a proper framework for "loose concepts", that connote the nonexistence of sharp boundaries.

2. Vagueness

Vagueness is a word of modern science. John Locke (1632-1704) used this word in his *Essay on human understanding* (1689) when he complained about the "vague and insignificant Forms of Speech" and in the French translation (1700) the French word "vague" is used for the English word "loose". Nevertheless the word "vague" became not a technical term in philosophy and logic during the 18th and 19th century. In the young 20th century, when the German philosopher and mathematician Gottlob Frege (1848-1925) published his *Basic Laws of Arithmetic*, he called for concepts with sharp borders because otherwise we could break logic rules and, moreover, our drawing of conclusions could be false [1]. Thus, Frege specified vagueness as a particular phenomenon, and he influenced other logicians, mathematicians, and philosophers, Bertrand Russell (1872-1970) who lectured on *Vagueness* in 1923 [2], and the two members of the Lvov-Warsaw School, Tadeusz Kortabiński (1886-1981) and Kazimierz Adjukiewicz (1890-1963) who defined "vagueness" as the existence of fluent boundaries [3, 4]. For Russell vagueness was "a conception, applicable to every kind of representation – for example, a photograph" and such "a representation is vague when the relation of the representing system to the represented system is not one-one but one-many" [2].

In the USA, Charles Sanders Peirce (1883-1914) gave – independent from Frege – a definition of vagueness as a new phenomenon in 1901, when he described the concept "vague" in a dictionary [5] and in 1905 in *Issues of Pragmaticism* [6]. For Peirce "a proposition is vague when there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed by the proposition" [5].

In 1934, Max Black (1909-1988), a philosopher and mathematician at the Queen's College in London, investigated vagueness in logic and mathematics. He connected Russell's and Peirce's approach, and he distinguished vagueness from ambiguity, generality, and indeterminacy: "The "vagueness of a term is shown by producing "borderline cases", i.e., individuals to which it seems impossible either to apply or not to apply the term [7]. Later, Black referred vagueness a "loose concept" [8]

Figure 1: C. S. Peirce, B. Russell, M. Black.

In Poland, Kortabiński defined that a concept for a property is vague (Polish: chwiejne) if the property may be the case by grades [3], and Adjukiewicz gave the definition: "a term is vague if and only if its use in a context decidable … will make the context undecidable in virtue of those [language] rules" [4].

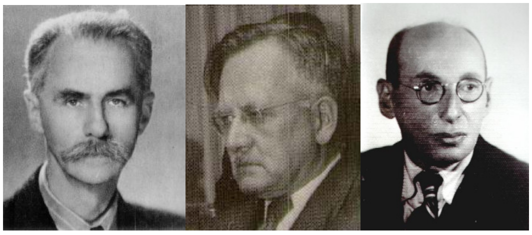

Figure 2: T. Kortabiński, K. Adjukiewicz, L. Fleck.

Ludwik Fleck (1896-1961), a young physician and philosopher was close to the Polish school of logic, but he was an opponent of the view that medical diagnoses are the result of strong logical reasoning. He thought that elements of medical knowledge, symptoms and diseases, are essentially indeterminate and that physicians count on their intuition instead on logical consequences to deduce from the patient's data to a disease. In 1927, Fleck published *Some specific features of the medical way of thinking* [9], where he emphasized that "a medical man studies precisely the atypical, abnormal, morbid phenomena. And it is evident that he finds on this road a great wealth and range of individuality of these phenomena which form a great number without distinctly delimited units, and abounding in transitional, boundary states."

Apparently, it is very difficult to define sharp borders between various symptoms in the set of all symptoms and between various diseases in the set of diseases, respectively. Rather we can observe smooth transitions from one entity to the other and perhaps a very small variation might be the reason that a medical doctor diagnoses a patient with disease x instead of disease y.

Fleck considered the "space of phenomena of disease" and he realized that there are neither boundaries in a continuum of phenomena of diseases and between what is diseased and what is healthy.

3. Haziness

A former member of the Vienna circle and a professor of geometry, Karl Menger (1902-1985), immigrated into the USA in 1937. He became a professor in Chicago and he was intended to generalize the theory of metric spaces towards probabilistic concepts. Thus, he introduced a "statistical metric", i. e. "a set S such that with each two elements („points") p and q of S a probability function … is associated" … that "is the *distance function* of p and q that bears the meaning of the *probability* that the points p and q have a distance ≤ x." [10] In this paper Menger introduced the also the concept of a *triangular norm* (*T-norm*).

10 years later, Menger looked for a tool to deal with elements of the physical continuum that is different from the mathematical continuum because these elements can be indistinguishable but not identical. Regarding A, B, and C as elements of a continuum, he referred to Henri Poincarés (1854-1912) claim "that only in the mathematical continuum the equalities $A = B$ and $B = C$ imply the equality $A = C$. In the observable physical continuum, »equal« means »indistinguishable«, and $A = B$ and $B = C$ by no means imply $A = C$. »The raw result of experience may be expressed by the relation $A = B$, $B = C$, $A < C$ which may be regarded as the formula for the physical continuum.« According to Poincaré, physical equality is a non-transitive relation." [11] Menger suggested a realistic description of the equality of elements in the physical continuum by associating with each pair (A, B) of these elements the probability of finding A and B indistinguishable. To solve "Poincarés paradox" he used his concept of probabilistic relations and geometry, and he introduced a concept of "equality sets". [11].

In 1951, when Menger became a visiting lecturer in Paris, he proposed to replace the set theory's element-relation "∈" between each object x in the universe of discourse U and a set A by the probability of x belonging to A. In contrast to ordinary sets he named these entities by "ensembles flous" and in English by "hazy sets". [12]

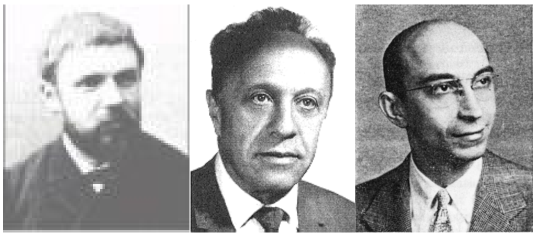

Figure 3: J. H. Poincaré K. Menger, L. A. Zadeh

Resuming his work on statistic metrics, probabilistic distances, indistinguishableness of elements, and ensembles flous, Menger believed that it would be important in geometry to combine these concepts with the concept "of lumps, which can be more easily identified and distinguished than points. Moreover, lumps admit an intermediate stage between indistinguishability and apartness, namely that of overlapping. "Lumps would not be point sets; nor would they reflect circumscribed figures such as ellipsoids. They would rather be in mutual probabilistic relations of overlapping and apartness, from which a metric would have to be developed." [13]

Menger never envisaged a mathematical theory of loose concepts that differs from probability theory. He compared his "micro geometry" with the theory of fuzzy sets in 1966: "In a slightly different terminology, this idea was recently expressed by Bellman, Kalaba and Zadeh under the name fuzzy set. (These authors speak of the degree rather than the probability of an element belonging to a set.)" [13] Menger did not see that this "slightly difference" between "degrees" (fuzziness) and "probabilities" is not only a difference in terminology but in the concepts' meanings.

4. Fuzziness

Even though this article by Bellman, Kalaba, and Zadeh was published in 1966 [14], the text was written by Zadeh in 1964 and printed for the first time as a memorandum for the RAND-Corporation [15]. Bellman was a good friend of Zadeh and in summer of 1964 they both discussed Zadehs idea of fuzzy sets as "a notion which extends the concept of membership in a set to situations in which there are many, possibly a continuum of, grades of membership." [15] In that time, Zadeh was a well-known scientist, who established system theoretical concepts in electrical engineering. In spring of 1965, Zadeh gave a talk on "A New View on System Theory". This "new view" dealt with the concepts of the theory of fuzzy sets and Zadeh explained that "these concepts relate to situations in which the source of imprecision is not a random variable or a stochastic process but rather a class or classes which do not possess sharply defined boundaries." [16] The later published manuscript is entitled with *Fuzzy Sets and Systems* [16].

5. Conclusion

The theory of fuzzy sets is a mathematical theory to deal with vagueness and other loose concepts, if the meaning of these concepts is the absence of strict boundaries. It seems that "vagueness", as it is used in philosophy and logic since the 20th century, may be formalized by fuzzy sets, whereas "haziness" like other scientific concepts, e.g. indeterminacy, is a concept that needs formalization by probability theory and statistics.

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